A METHOD FOR COMPARING SURFACE TO AIR GUIDED MISSILE SYSTEMS FOR THE DEFENSE OF NAVAL SURFACE UNITS

F. H. BURNHAM

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A METHOD FOR COMPARING SURFACE TO AIR GUIDED MISSILE SYSTEMS FOR THE DEFENSE OF NAVAL SURFACE UNITS

by

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Lieutenant Commander, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE

United States Naval Postgraduate School
Monterey, California

1 9 5 4

This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE

from the

United States Naval Postgraduate School

PREFACE

This paper considers certain problems which will arise in connection with the employment of surface launched guided missiles for anti-aircraft defense of naval surface units. It is stated here, once for all, that for the sake of brevity, such expressions as "guided missile team" and "guided missile system", are always used with the meaning: surface launched guided missiles for anti-aircraft defense of naval surface units.

The great progress made in the development of guided missiles in the past decade makes it appear highly probable that there will be available, in the near future, many guided missile systems suitable for installation on various kinds of ships. From a search of the available literature, it appears that there has been no extensive study made of how ships with such installations can be assigned most effectively to the defense of naval surface groups (convoys, task forces, independents) requiring such protection. Accordingly, this paper sets forth the idea that there may well be a number of different kinds of ship teams for providing this type of protection. The concept of a team is that of one or more ships, possessing capabilities for launching and/or guidance and possibly performing some part of the target detection function. Each type of team would be based upon a particular guided missile system.

The team concept is itself not essentially new, but has suggested itself in other cases where the weapon system has had a specific rather than a multi-purpose role, (e.g. hunter-killer teams, ASW teams, night fighter and night attack teams).

With the guided missile team concept, it becomes necessary to determine the criteria on the basis of which such teams should be assigned to various surface groups requiring this kind of protection. It is with the determination of these criteria that this paper is primarily concerned.

In determining such criteria, it is essential that certain striking contrasts between guided missile defense and conventional anti-aircraft defense of naval surface units be recognized, namely:

(1) The cost of guided missiles as compared to the cost of conventional anti-aircraft weapons.

(2) The relatively limited numbers of guided missiles which can be carried by a team as compared to the essentially unlimited numbers of expendable rounds of conventional anti-aircraft ordnance which can be carried by a group of ships. This aspect of the problem is similar to that of submarines in the use of their torpedoes.

This study, undertaken at the United States Naval Postgraduate School during the latter half of academic year 1954, attempts to provide a quantitative analysis of the defensive potential of several guided missile teams. Particular emphasis has been placed upon the problem of selecting an optimal team for the anti-aircraft defense of naval surface units operating without the benefit of air support, since the writer believes that guided missiles for anti-aircraft defense will make their greatest single contribution under such circumstances.

The writer is indebted to Dr. T. E. Oberbeck and Dr. C. C. Torrance of the United States Postgraduate School Staff. Their suggestions and help in the preparation of this paper are sincerely appreciated.

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CHAPTER I
INTRODUCTION

1. Summary

This study examines the problem of selecting an optimal guided missile team for the defense of a naval surface unit against aircraft attack when several teams are available and the characteristics of each team are known.

A measure of effectiveness, for making quantitative comparisons of such teams, is defined and its application is demonstrated for two conditions under which these teams may be expected to operate. These operating conditions are defined as follows:

<u>Operating condition one:</u> An upper limit of the expected damage per attack to the protected unit is specified.

<u>Operating condition two:</u> No upper limit of the expected damage is specified, but it is desirable that the ratio of enemy loss to our loss be a maximum.

Methods are formulated for calculating the values of the parameters appearing in the measure.

For the sake of concreteness, the measure of effectiveness is applied to make a comparison of three hypothetical teams which could be used for the defense of a convoy.

Although this measure was defined without consideration of contributions to the defensive effort by friendly air-craft and anti-aircraft guns, it can be applied with minor modifications to cases where these weapons are an integral part of the defense.

2. The situation

The problem of assigning the optimal guided missile team to the defense of a naval surface unit is predicated upon the following assumed situation:

A Fleet or Force Commander must provide anti-aircraft protection for a group of surface vessels which are expected to experience attacks by enemy aircraft during a voyage. The assignment of aircraft carriers for this purpose is not feasible. The Commander has at his disposal several guided missile teams which have the capability of defending against aircraft attack. The characteristics of each of the available teams are known. The team which the Commander assigns must operate under one of two conditions, namely:

(a) Because of the military situation, the expected damage to the defended unit per attack must not exceed a certain value.

(b) The expected damage to the defended unit per attack is not limited, but it is desired that the ratio of enemy losses to our losses be a maximum.

3. The approach to the problem

The problem is approached by defining a measure of effectiveness for making quantitative comparisons of the teams available for assignment. An attempt has been made to include in the measure only those parameters which are essential to a satisfactory comparison. Techniques are then formulated for determining the values of the parameters used in the measure of effectiveness. Since the values of some of the parameters in the measure are dependent upon the characteristics of the attack to be defended against, consideration is then given to the problem of determining

the expected values of the variables which characterize an attack. Finally, the use of the measure is examined for the comparison of teams under each of the operating conditions described in part two of this introduction, since it is believed that these are the two general conditions under which teams must be chosen.

4. Assumptions

The following assumptions have been made and apply throughout:

(1) The primary mission of any naval surface unit defense is the immediate defense of the unit. For this reason no targets will be engaged after they have passed their weapon release range.

(2) A ship may have several missile control stations and each control station may control several missiles simultaneously; however, one missile control station can engage only one target at a time. This assumption limits the number of simultaneously engaged targets to the number of available control stations.

(3) The number of control stations assigned to a team will be determined by the estimated number of aircraft in the expected attack and the characteristics of the team selected for defense.

(4) All attacking aircraft are detected by a certain minimum range.

CHAPTER II

THE MEASURE OF EFFECTIVENESS AND TARGET CONSIDERATION

1. The measure of effectiveness

In choosing a measure of effectiveness for comparing the various guided missile teams, the following factors are considered:

(1) The cost of a guided missile is much greater than that of any expendable weapon ever before used for defensive purposes. This cost is so great that it can no longer be ignored as a factor when planning a defense.

(2) The great increase in the lethality of weapons in the past decade makes it highly desirable that defenses be tightened. The alternative is to accept greater losses in the protected unit.

(3) A defense may be tightened by increasing the volume of fire. However, increasing the volume of fire, when the defensive weapon is a guided missile (the cost of which may be an appreciable fraction of the cost of the unit being protected) can easily make the cost of the defense more than the value of the protected unit. In such cases it may be necessary to strike a compromise between the level of defense achievable and the expected damage to the protected unit in order to minimize the total cost (i.e. the amount expended for defense plus the amount necessary to repair or replace the damage to the protected unit).

(4) When the operating condition is such that minimization of damage to the protected unit is the primary objective of the defense, the cost of the defense becomes a secondary consideration and a measure of effectiveness which stresses

probability of a kill is indicated. Alternatively, when the operating condition allows a choice of the level of defense, total cost to us may become the fundamental factor in the measure of effectiveness.

The measure of effectiveness has therefore been chosen as a ratio of the two fundamental comparison criteria as follows:

$$\text{M.E.} = \frac{\text{Probability that an attacking aircraft is destroyed}}{\text{Total expected cost to us}}$$

Definition of the measure of effectiveness as a ratio of the two fundamental comparison criteria insures consideration of both, regardless of the operating condition under which the team is to be chosen. This form of the measure of effectiveness also insures that it remain a sensitive instrument for comparison, regardless of the characteristics of the teams to be compared.

Since the value of most material things can be conveniently measured in terms of dollars, this unit is taken as a measure of the total cost to us. By putting the expression for M.E. into a mathematical form, a quantitative estimate can be made of the relative worth of several teams.

The mathematical form of the measure of effectiveness is,

$$\text{M.E.} = \frac{\sum_{r=x}^{n} \binom{n}{r} (P_h)^r (1-P_h)^{n-r}}{nC_m + \sum_{r=0}^{x-1} \binom{n}{r} (P_h)^r (1-P_h)^{n-r} D(L,v)} \qquad \text{(I)}$$

The numerator of this expression is the probability that a detected and engaged aircraft be destroyed. In the numerator, the symbol P_h represents the probability that any single missile fired hits the target, the symbol x represents the minimum number of hits necessary for destruction of the target, and the symbol n represents the number of missiles fired at the target.

The denominator of this expression is the total expected cost to us and is the sum of two factors. These are: (1) the direct cost of the missiles expended in attempting to destroy the attacking aircraft and (2) the cost of repairing the damage or replacing the ships sunk by the attacking aircraft if it penetrates the defense. In the denominator the symbol C_m represents the cost of the missile in dollars, the symbol D(l, v) represents a damage factor, which is a function of the lethality of the enemy weapon and the vulnerability of the class of vessel being protected, expressed in dollars, and the rest of the second term is the complement of the numerator, i.e. the probability that the attacking aircraft penetrates the defense.

When the characteristics of the missile are such that only one hit is required for destruction of the target, expression (I) reduces to the form,

$$\text{M.E.} = \frac{1 - (1 - P_h)^n}{nC_m + (1 - P_h)^n \; D(L, v)} \tag{II}$$

It must be emphasized that in those cases where the parameter x assumes values greater than one the denominator of expression (I) represents the maximum expected cost to us for the following reason: There will be an expected number of aircraft which will survive even thought having been hit one or more times but less than x times. These aircraft may have been damaged enough by the hits sustained to lessen or even void their potential lethality.

2. Target considerations

Any possible aircraft attack is determined by specifying the values of the following essential variables:

(1) Aircraft velocity

(2) Aircraft altitude

(3) Aircraft armament

(4) Armament release range

(5) Number of aircraft

(6) Attack procedures

It is, of course, impossible to specify in advance the values which any of the variables of an attack will assume, since they may take on any value, within specific limits, at the discretion of the enemy. The best that can be done is to determine the expected values of these variables, calculated on the basis of an assumed frequency of occurrence. The method of arriving at the values of the essential variables for the expected attack is described in Appendix A.

The computation necessary to determine the expected values of the essential variables can be somewhat simplified by grouping the possible enemy attacks into two categories. These are:

(1) Attacks employing armament having its own propulsion. An example of such armament is an air-to-surface guided missile whose release range is essentially independent of releasing aircraft speed and altitude.

(2) Attacks employing armament without its own propulsion. An example of such armament is a conventional bomb or glide

bomb whose release range is totally dependent upon releasing aircraft speed and altitude.

Attacks falling into category one above can be fully specified only by assigning values to all the listed essential variables. Attacks falling in category two above can be fully specified by assigning values to the independent variables (1), (2), (3), and (5), the value of variable (4) being completely determined by the values of variables (1) and (2). To specify the expected values for the variables of an attack, the methods of Appendix A may be used if data on past attacks are available. If such data are not available, the values for these variables must be estimated from intelligence information or some other means.

CHAPTER III

THE APPLICATION OF THE MEASURE TO A HYPOTHETICAL PROBLEM

There are two general cases into which a typical problem may be classified.

Case (1) Military necessity dictates that the expected damage to our forces must not exceed a certain fixed level. An example of such a case would be the delivery by convoy of items which have suddenly become critical in the first stages of an invasion. The loss of more than a certain number of ships from this convoy would jeopardize the success of the invasion. Such conditions dictate that the cost of the weapon is insignificant, and they will in general be the result of tactical considerations.

Case (2) The military situation does not dictate that expected losses be held to any fixed minimum, but it is desirable that, over an extended period, the mission be accomplished as economically as possible. An example of such a case would be the build up of forces by convoy preparatory to mounting an invasion. Such cases will usually evolve from strategic rather than tactical considerations.

Certain aspects of the method are applicable to both cases. The program will be to demonstrate the method to the point where it no longer applies equally to both cases, at which juncture each case will be considered separately.

For purposes of demonstration, let us assume that the Commander upon whom falls the responsibility of assigning guided missile teams for anti-aircraft protection of convoys has three basic types of teams

available, with characteristics as follows:

	Team 1	Team 2	Team 3
R_m	20 miles	40 miles	80 miles
r	6/min	1.5/min	.5/min
T_f	3 min	4 min	5 min
C_m	$5K	$10K	$15K
x	2	1	1
P_h	.5	.6	.7
V_m	1200 kts	1000 kts	700 kts

Information on 40 recent attacks is available and is as follows:

V_t	f
200	1
300	5
400	7
500	25
600	2

h_t	f
100 ft	5
500	18
5000	7
10,000	6
20,000	3
30,000	1

N	f
1	5
2	9
3	2
4	16
6	4
8	4

R_s	f
1 mile	20
2	8
5	7
20	4
40	1

L	f
2	10
4	16
6	8
10	4
20	2

From the foregoing information, the essential variables of the expected attack can be computed, by the method of Appendix A where

no trend or distribution is recognizable, as:

$\overline{V}_t$ = 455 kts

$\overline{h}_t$ = 4,862 ft

$\overline{N}$ = 3.75

$\overline{R}_s$ = 4.75 miles

$\overline{L}$ = 5.3 500 lb. bombs

Let us further assume that for the convoy to be protected and the expected value of L, the value of $D(L,v) = .5 \times 10^6$ dollars.

The limiting value of n is now calculated for each of the three teams to be compared. This will be illustrated in detail for team 1, the value for the other two being calculated in the same manner. Using expression (2) of Appendix B, and assuming a detection range of 100 miles, $T_d = 9\frac{1}{2}$ minutes. Since T_f for this team is less than the computed value of T_d enter figure (2) with $T_d = 9\frac{1}{2}$ and $V_t = 455$ obtaining $R_f = 28$ miles. This firing range assures $R_i = 20$ miles. Now, by combining expression (5) of Appendix B with the assumption that we do not desire to engage the target after he releases his armament, n = 12. By the same procedure n = 7, 2 for teams 2 and 3 respectively..

From this point on the procedure differs according to whether or not a limit to the expected damage is set.

Case I <u>The case of limited expected damage</u>

When the limit of expected damage is specified, the controlling criterion is whether or not a team has the capability of providing a value of

$$\sum_{r=0}^{x-1} \binom{n}{r} (P_h)^r (q_h)^{n-r} D(L,v) \lesseqgtr M$$

where M is the designated upper limit of expected damage per attacking aircraft. Any team which does not have this capability can be immediately dropped from consideration, and those remaining can be compared to choose the optimum one. To determine which teams have the capability, note that the value of $D(L, v)$ is determined by the value calculated for $\bar{L}$ of the expected attack, and the composition of the convoy. Therefore the value that n must assume to meet the imposed damage restriction can be compared to the previously computed limiting values for n as a means of rejecting those teams which cannot meet the imposed damage restriction. This value that n must assume can be arrived at by means of graphs, of which figures (3)a and (3)b are typical, by selecting the next larger integral value of n for which $1 - P_k \leq \frac{M}{D(\bar{L}, v)}$.

Comparing the previously computed limiting values of n with the required values of n, it is seen that team 3 cannot meet the requirements. This is not what one would intuitively expect, inasmuch as team 3 has the greatest hit probability, the longest range, and a minimum number of hits necessary for a kill.

It now remains to compare the two systems which have the required minimum value of n to resolve the problem of which of these two is optimum under these conditions. Since both have the ability to hold the damage to the required minimum, the problem of evaluating these two to determine the optimum degenerates to that described in the next case with one restriction. This restriction is that the minimum value used for n must be greater than or equal to that determined as the value which n must have to meet the imposed damage restriction.

This completes the procedure to be used when an upper limit of damage to our forces is specified.

Case II <u>The case where no upper limit of damage is specified.</u>

In the case where no limit of damage is prescribed, the optimal team will be the one which causes expression I or II to assume the greatest value. Solving these expressions by the usual mathematical process for maximization leads to a transcendental equation which is most easily solved by graphical methods. For this reason, a graphical method of using overlays has been devised from which the value of n which maximizes the expression for a particular team, and the quantitative value assigned the team for this maximizing value of n can be readily calculated. The process is as follows:

$$\log ME = \log P_k - \log \left[nC_m + (1 - P_k)\, D(L, v) \right]$$

Now if a curve of $\log P_k$ vs. n is plotted and a curve of $\log \left[nC_m + (1 - P_k)\, D(L, v) \right]$ vs. n is plotted on the same coordinates, $\log ME$ will be a maximum at that value of n where the ordinate between the curves is a maximum. It follows that when $\log ME$ is a maximum, ME is a maximum; therefore the quantitative measure of the team, when the optimal number of missiles is fired, is obtained by taking the antilog of the maximum ordinate between the curves.

By plotting a family of curves of $\log P_k$ vs. n with P_h a parameter, for each value of x, a basic series of underlays can be made. Two typical such underlays are presented as figures (4)a and (4)b. Overlays can be made by plotting $\log \left[nC_m + (1 - P_k)\, D(L, v) \right]$ vs. n with P_h a parameter for various ratios of C_m to $D(L, v)$, one set of

overlays for each value of x being necessary. Partial overlays covering the range of values specified in this demonstration problem are included as figures (5)a, (5)b, and (5)c .

The comparison of the three teams is made as follows:

(a) For the team to be compared, select an underlay plotted for the appropriate value of x . For team 1 this underlay is figure (4)a.

(b) Determine the ratio of C_m to $D(L, v)$ and select an overlay for that ratio and the same value of x as the underlay. For team 1 this overlay is figure (5)a.

(c) Superimpose the overlay on the underlay and read that value of n which has the largest value of the ordinate between the curves, plotted for the applicable value of P_h .

(d) If this value of n is greater than the limiting value of n determined by means of Appendix B, take the value of the ordinate between the curves at the limiting value of n . The antilog of this quantity is the quantitative measure of the team.

(e) If the value of n , where the maximum ordinate between the curves appears, is less than the limiting value of n , take the value of the ordinate between the curves at the next higher or next lower integral value of n , whichever is greater. The antilog of this value is the quantitative measure of the team.

(f) The team which has the greatest quantitative value as a result of this procedure is then the optimum team to use under these conditions.

Using the above procedure, the following results are obtained:

	Optimum value of n	Comparison factor
Team 1	7	6.46
Team 2	4	7.55
Team 3	3(limiting value=2)	6.22(for limiting value)

The following statement can therefore be made: Using the procedure described herein the optimal number of missiles to be fired under the conditions of this problem is as indicated in the above table, and providing the optimal number of missiles is used, team 2 is better than team 1 by a factor of 1.17 and better than team 3 by a factor of 1.21.

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APPENDIX A

Techniques for Assigning Values to the Essential Variables of an Attack.

The essential variables of any attack by aircraft have been listed in Chapter II of this study. The purpose of this appendix is to discuss means of determining the values to be assigned to these variables.

There are three conceivable ways in which the values assigned to the variables of an attack may be chosen. These ways may be labeled:

(1) Outright guessing

(2) Game theoretic methods

(3) Application of Statistical methods.

At the outbreak of hostilities data on past attacks will not be available and outright guessing or scientific guessing must be used. Outright guessing is the least desirable method but may be necessary. In this procedure a value is assigned to each of the variables on the basis of intelligence reports, our knowledge of our own weaknesses and any other factors which are considered applicable. Application of game theory may be a better procedure to use in the absence of data on past attacks if it is possible to play a meaningful two person game. Whether the use of game theory is possible, and the procedure to use if it is possible are considered beyond the scope of this paper. It is believed, nevertheless, that such methods may have application to this problem.

When data on past attacks are available, statistical methods may be employed. It is believed that statistical methods are to be preferred when their employment is possible. The use of statistical methods requires that two fundamental questions be answered immediately. Are the stochastic variables involved independent, and are these variables

essentially discrete or continuous in nature? The question of independence can be answered by stating that over the range to which the variables in question are limited they can be considered independent. The question of discrete or continuous nature can be answered by stating that, despite the fact that they might take on any value within certain limits, enemy doctrine and equipment pecularities will probably cause these variables to assume a discrete character. The variables of an attack are therefore assumed to be independent and discrete.

Data on past attacks is nothing more than a sample taken from a parent population. Certain inferences can be made concerning the parent population by examining the parameters associated with these samples. A trend might be indicated or a distribution which the enemy was using might be discovered. For example, the enemy may have elected to vary his attack altitude stochastically using a normal distribution of frequency. A sample should be examined for these possibilities. It is believed, however, that in reality the values that the variables of an attack assume will be primarily influenced by the situation at the time of the attack. Consequently, the most likely use to which data could be put would be the calculation of simple expected values for each of the variables.

Under the assumption of independence and discreteness, the expected values of each of the variables could then be computed from the expression

$$\bar{r} = \sum_{i=\bar{r}\,min}^{\bar{r}\,max} r_i \, f(r_i) \, ,$$

where r is a value which the variable has taken and f(r) is the relative frequency of occurrence associated with that value.

When it is known that the enemy is not employing powered armament the expected value of armament release range ($\overline{R}_s$) may be computed from the expression

$$\overline{R}_s = \left(\frac{2\,\overline{h}_t\, v_t^2}{g} + h_t^2 \right)^{1/2},$$

where $\overline{h}_t$ is the expected value of target altitude, $\overline{V}_t$ is the expected value of target velocity, and g is the force of gravity.

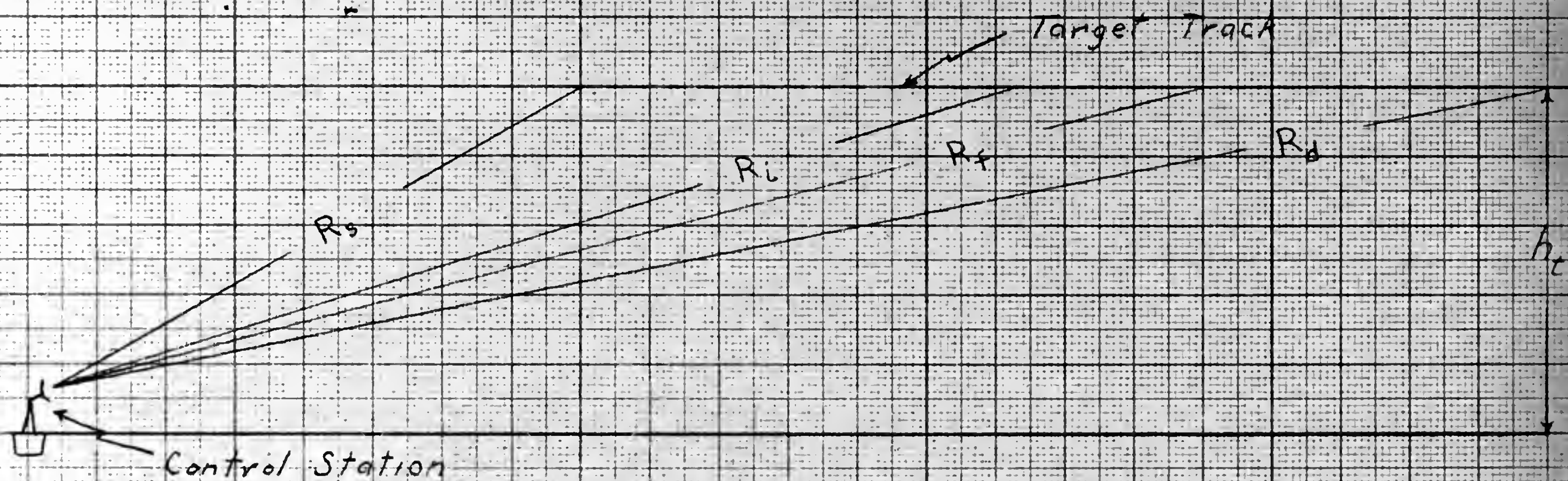


Figure 1

APPENDIX B

Determination of the Limiting Value for Number of of Missiles Fired

As stated in Chapter III of this study, it is necessary that a limiting value for the number of missiles that can be fired by one control station at one target be determined. The purpose of this appendix is to define a procedure for the determination of this value.

With reference to Figure (1) on the previous page, it can be shown that $R_f = (R_d^2 - 2(R_d^2 - h_t^2) V_t T_d + V_t^2 T_d^2)^{1/2}$ (1)

where R_f is the target range at time of missile firing. R_d is the range at which target is detected, h_t is target altitude, V_t is target velocity, and T_d is the elapsed time from time of target detection to time of missile firing. (T_d is the sum of T_f, the time it takes to ready the missile for firing, and T_w, the time a ready missile must be held while the target approaches missile range.) For radar detection ranges, h_t is insignificant and expression (1) reduces to $R_f \doteq R_d - V_t T_d$. Curves of R_f vs. V_t with T_d a parameter are plotted in Figure (2).

A question arises concerning the value of T_d to use in entering Figure (2), since T_d is a function of missile maximum range. It can be demonstrated that the necessary condition for the target range at intercept to be less than or equal to the missile maximum range is

$$T_d \geq \frac{R_d}{V_t} - R_m \left(\frac{1}{V_m} + \frac{1}{V_t} \right) \qquad (2)$$

where R_m is missile maximum range and V_m is missile velocity. Accordingly, for missiles whose T_f is less than T_d as computed by expression (2) enter Figure (2) with the computed value of T_d, otherwise

enter Figure (2) with a value for T_d equal to T_f. Negative values of T_d as computed from expression (2) merely indicate that missile range is not a factor which need be considered; therefore use the known value of T_f for T_d when entering Figure (2).

From a further consideration of Figure (1), it can be determined that

$$R_i = \frac{\left[R_t^2 - \left(\frac{h_t V_t}{V_m} \right)^2 \right]^{1/2}}{1 - \left(V_t / V_m \right)^2} - \frac{\left(R_f^2 - h_t^2 \right)^{1/2} V_t}{V_m^2 - V_t^2}, \qquad (3)$$

where R_i is the range of the target when the first missile fired intercepts.

It is required that all missiles that are fired intercept the target between range R_i and R_s, where R_s is the enemy weapon release range. This distance along the target path is $(R_i^2 - h_t^2) - (R_s^2 - h_t^2)^{1/2}$. Therefore, the time available for interception is

$$t = \frac{\left(R_i^2 - h_t^2 \right)^{1/2} - \left(R_s^2 - h_t^2 \right)^{1/2}}{V_t} \qquad (4)$$

But the number of missiles which can be fired is the product of the rate of fire of the system and the time available to fire, i.e.,

$$n = rt = \frac{r\left[\left(R_i^2 - h_t^2 \right)^{1/2} - \left(R_s^2 - h_t^2 \right)^{1/2} \right]}{V_t} \qquad (5)$$

This value for n is the limiting value which n may assume based upon the characteristics of the team and the values specified for the essential variables of the attack.

In most practical cases, the quantity h_t may be deleted from all the above expressions as insignificant.

LIST OF SYMBOLS

V_t	Target velocity
$\overline{V}_t$	Expected attack velocity
h_t	Target altitude
$\overline{h}_t$	Expected attack altitude
L	Lethality of enemy armament relative to 1-500 lb bomb
$\overline{L}$	Expected attack weapon lethality
N	Number of attacking aircraft per attack
$\overline{N}$	Expected number of attacking aircraft
R_s	Attack weapon release range
$\overline{R}_s$	Expected attatck weapon release range
R_i	Target slant range at which first missile launched makes interception.
R_f	Target slant range at time first missile is fired.
R_d	Slant range at which target is detected.
R_m	Missile maximum range
P_h	Single shot probability that missile reaches fuze actuation range, and fuze actuates.
x	Number of missiles which must reach fuze actuation range with fuze actuation to give a near certain probability of target destruction.
n	Number of missiles fired at one target.
v	Average surface vessel vulnerability to one 500 lb bomb. There will be a characteristic value for each major type of vessel.
C_m	Cost of one missile. This may be relative to the cost of the protected unit or the expected damage factor.
$D(L, v)$	Damage factor. A function of the vulnerability of the class being protected and the expected lethality of the attack weapon.

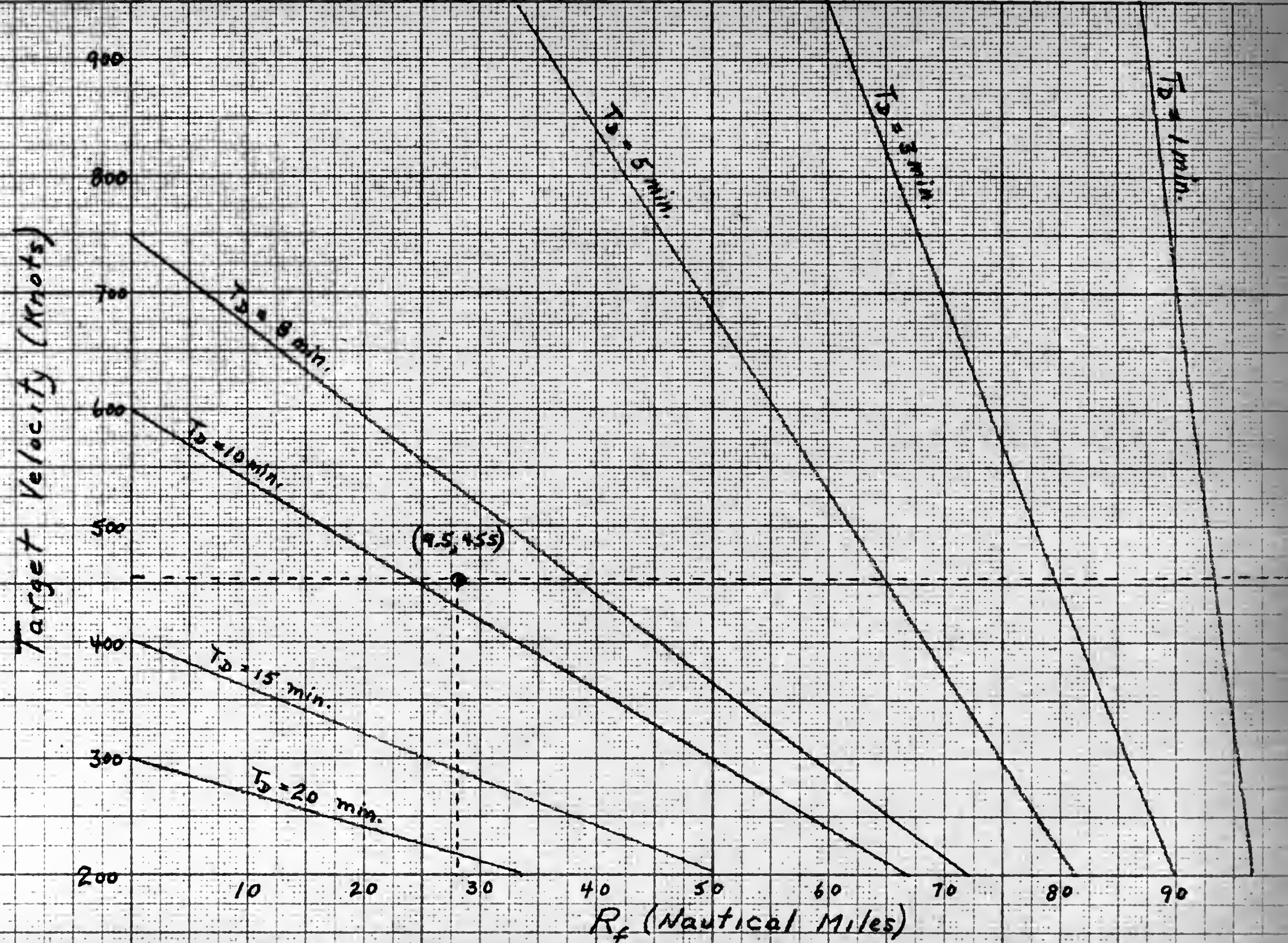
T_f Elapsed time between target detection and missile ready to fire.

T_w The time that a missile which is ready to fire is held, while target approaches missile maximum range.

T_d Dead time. The sum of T_f and T_w.

r Rate of fire

V_m Velocity of missile



T_D = 1 min.
T_D = 3 min.
T_D = 5 min.
T_D = 8 min.
T_D = 10 min.
T_D = 15 min.
T_D = 20 min.
(9.5, 455)
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80
90
Target Velocity (Knots)
R_f (Nautical Miles)

Figure 2

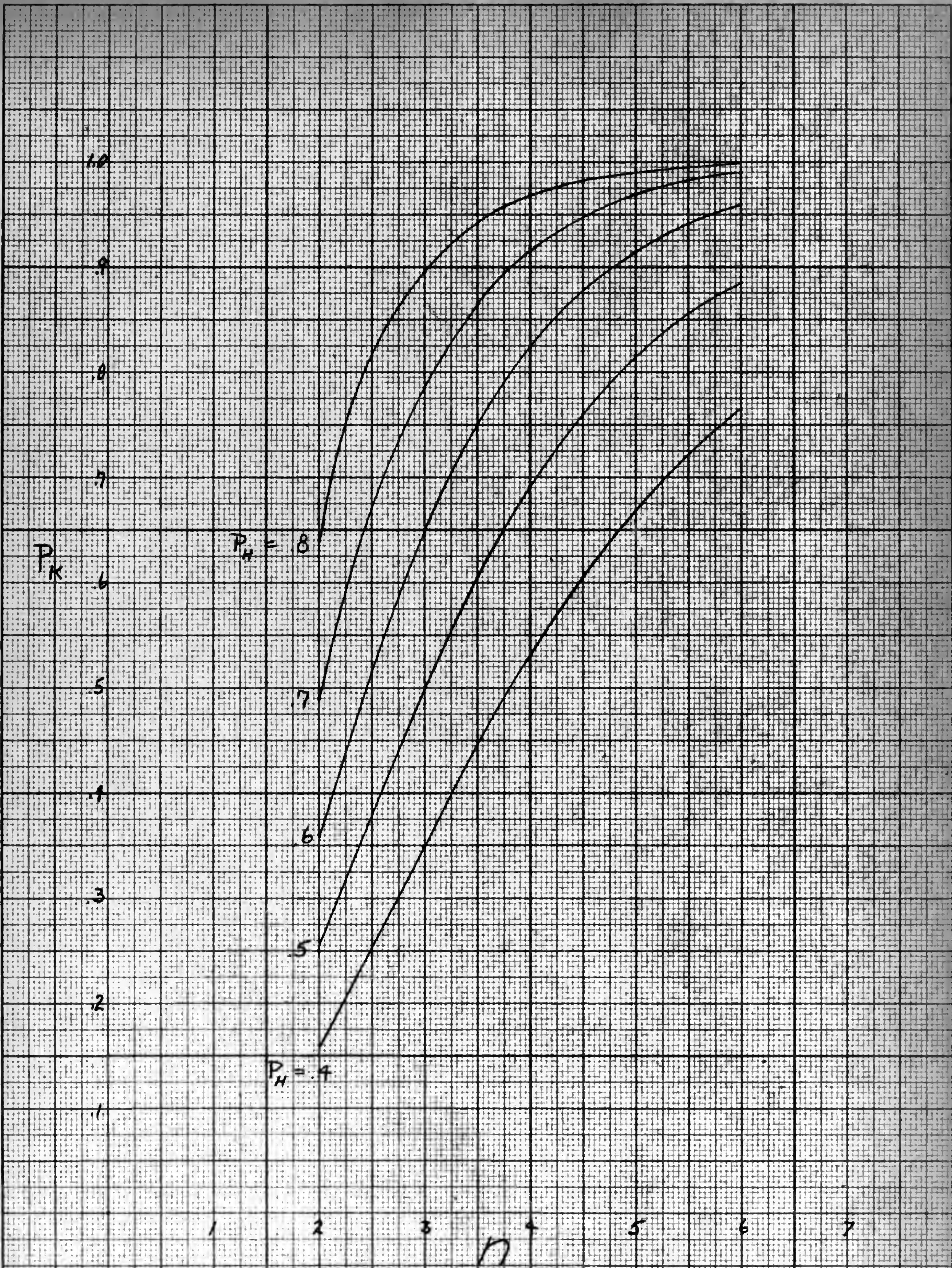

Figure 3(a)

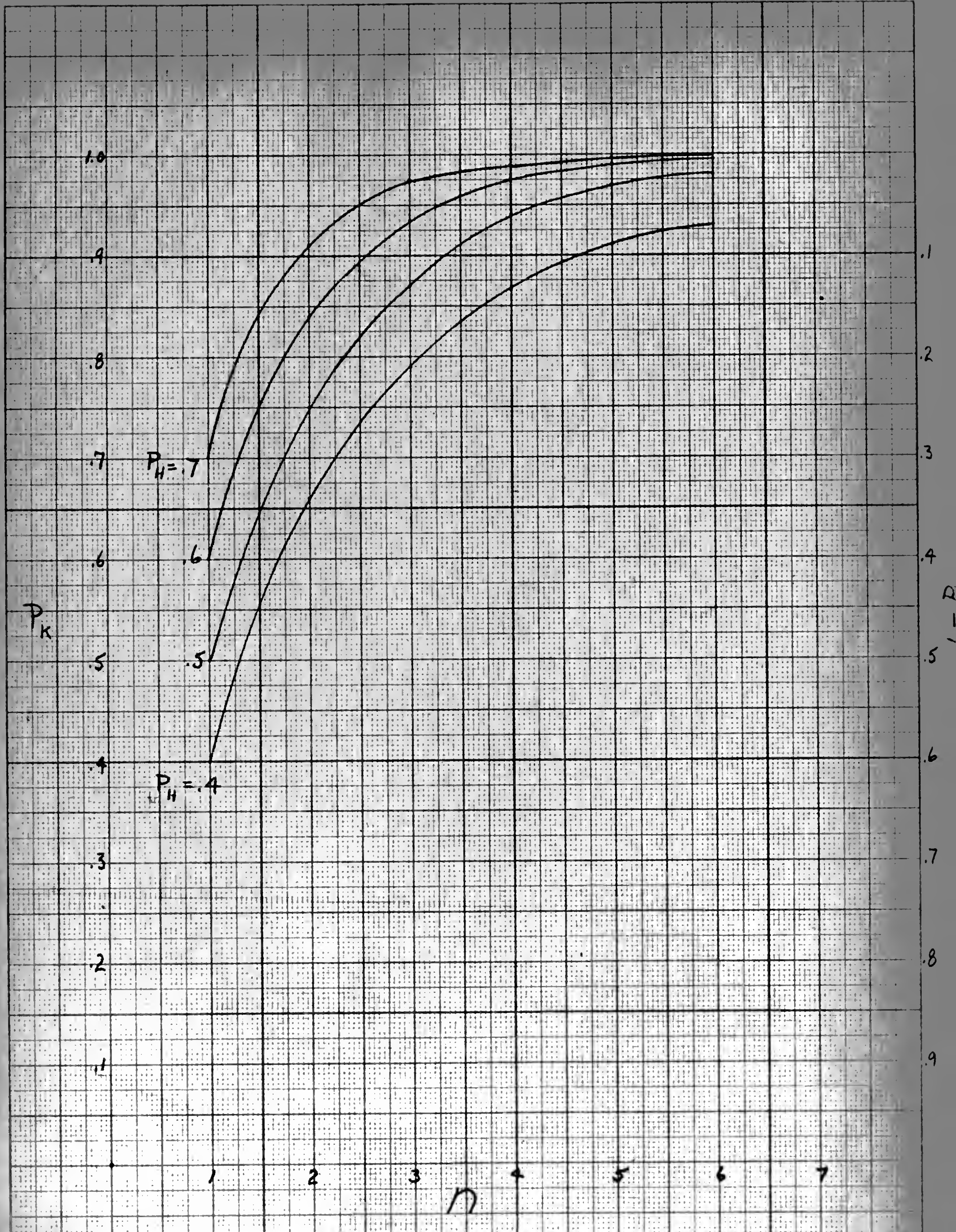

Figure 3(b)

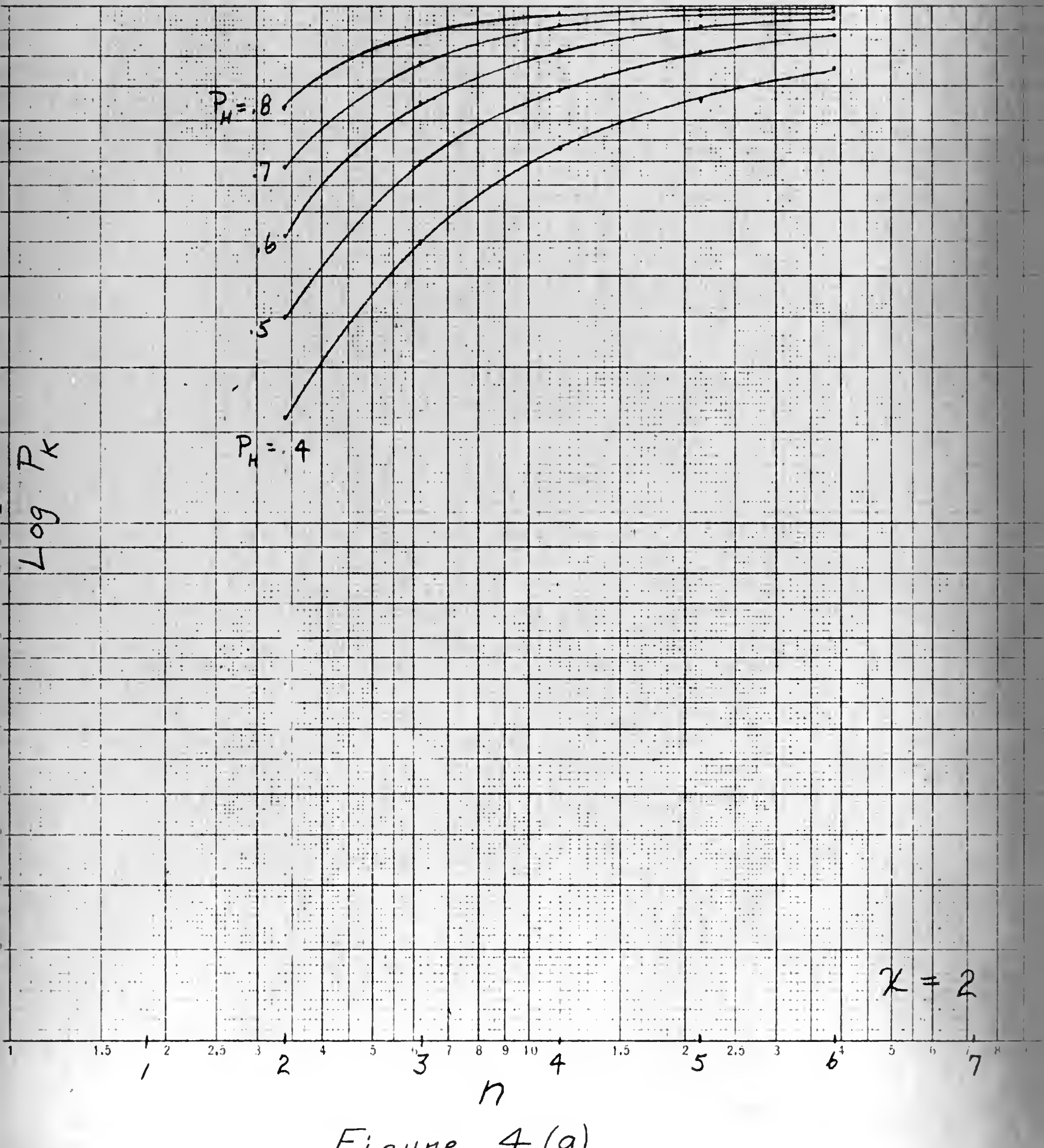

Figure 4 (a)

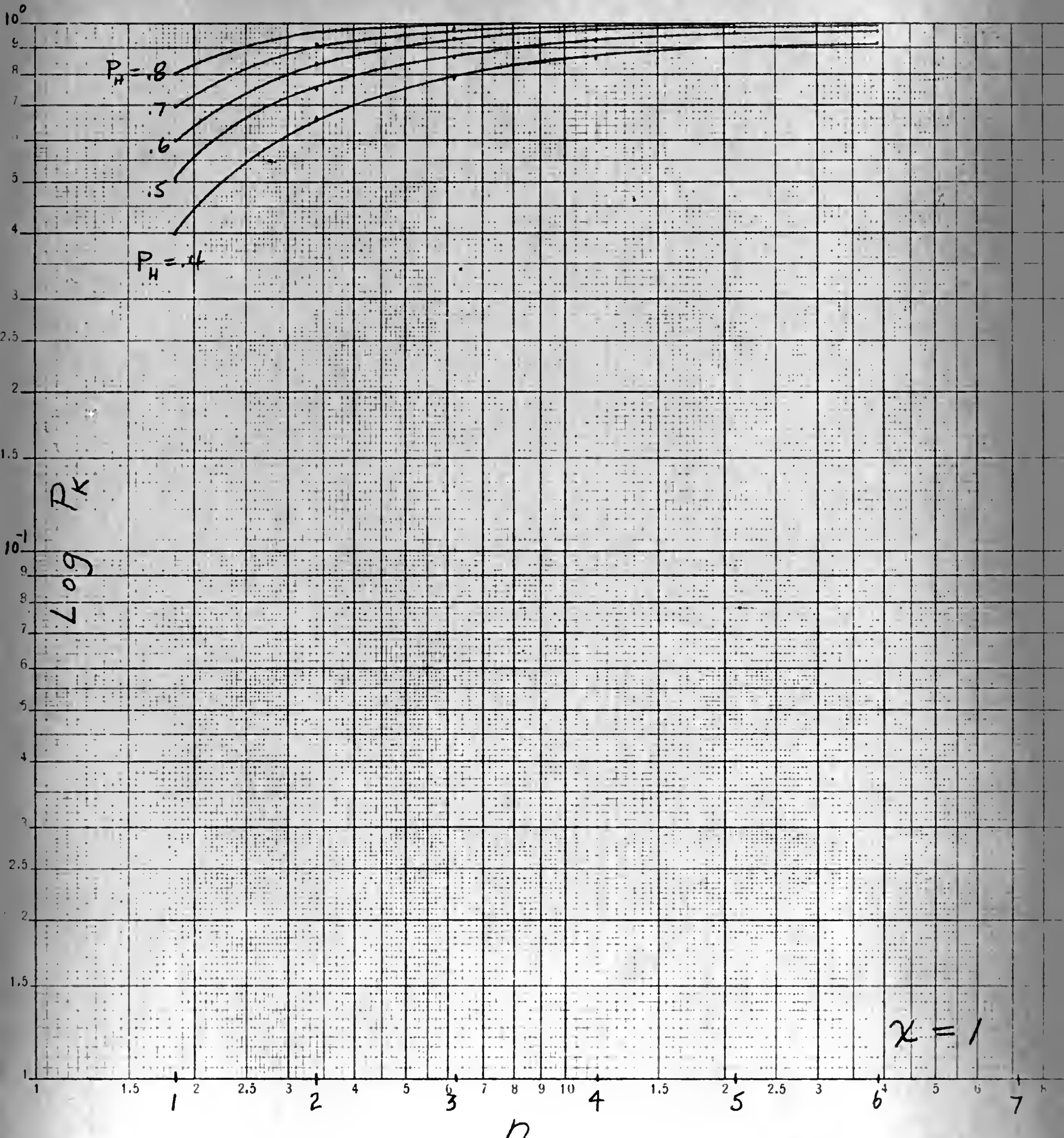

Figure 4 (b)

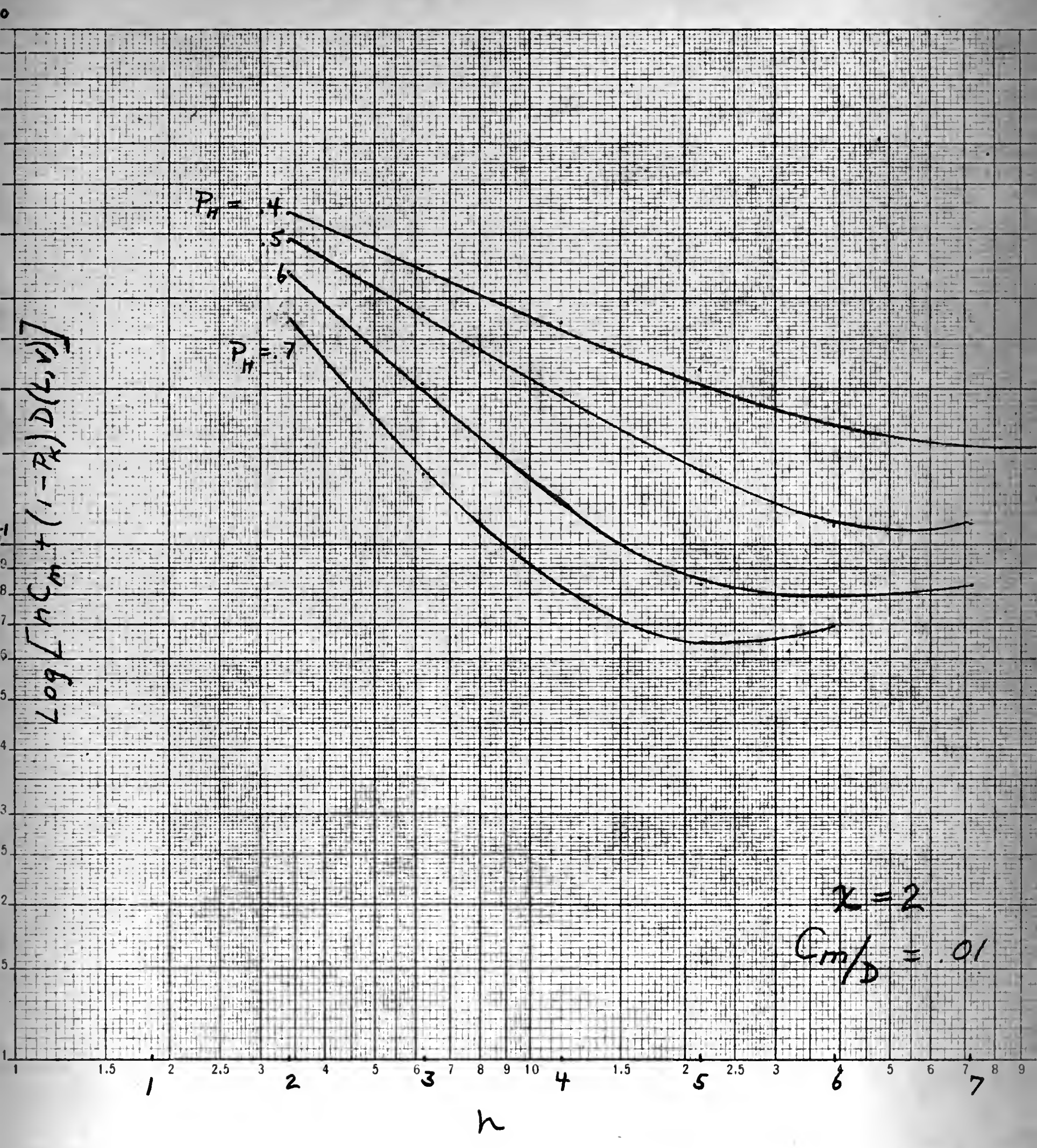

Figure 5(a)

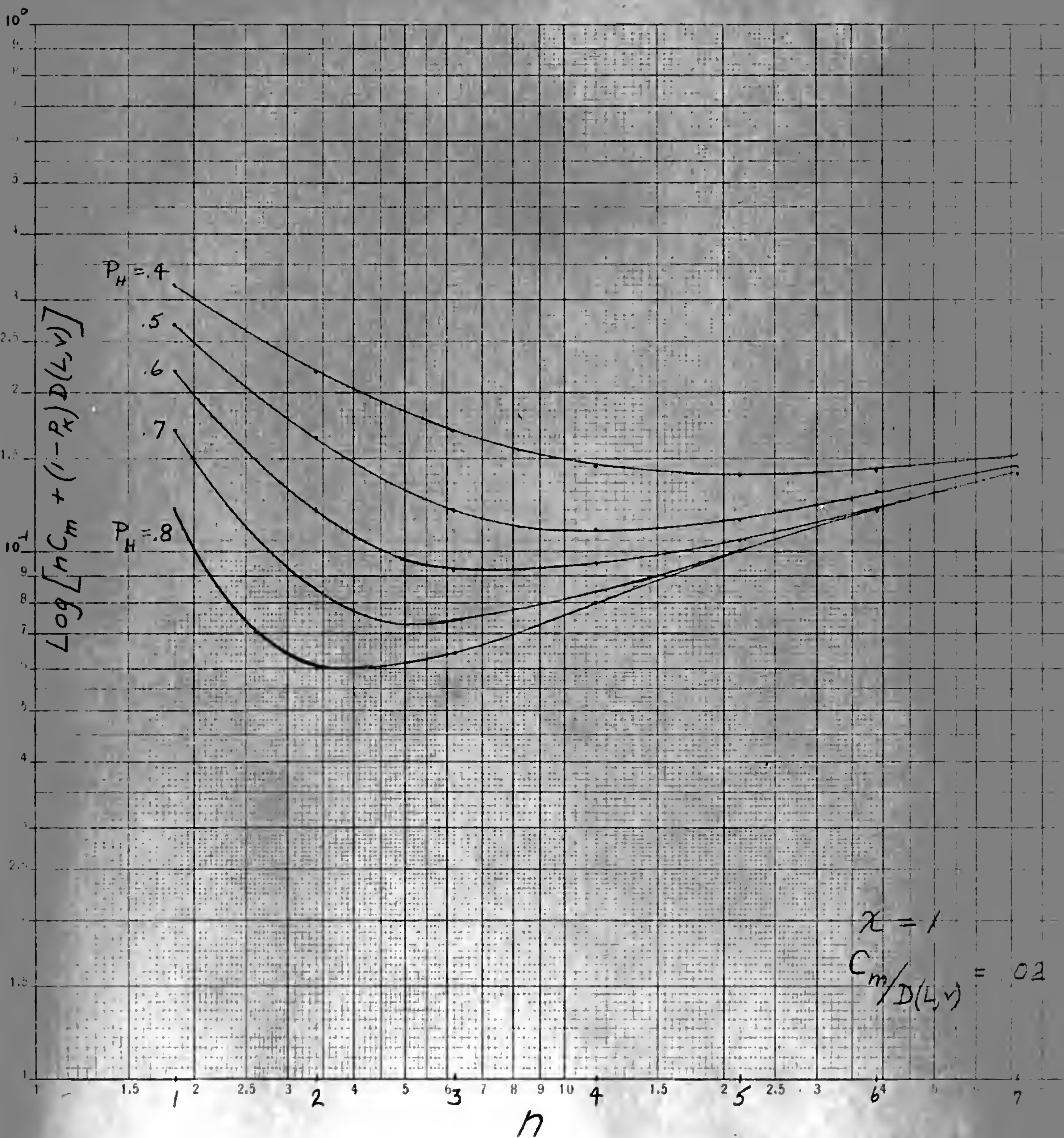
10^0
10^{-1}
$Log\left[nC_m + (1-P_K)D(L,V)\right]$
$P_H = .4$
.5
.6
.7
$P_H = .8$
$x = 1$
$C_m / D(L,V) = .02$
1
2
3
4
5
6
7
n

Figure 5(b)

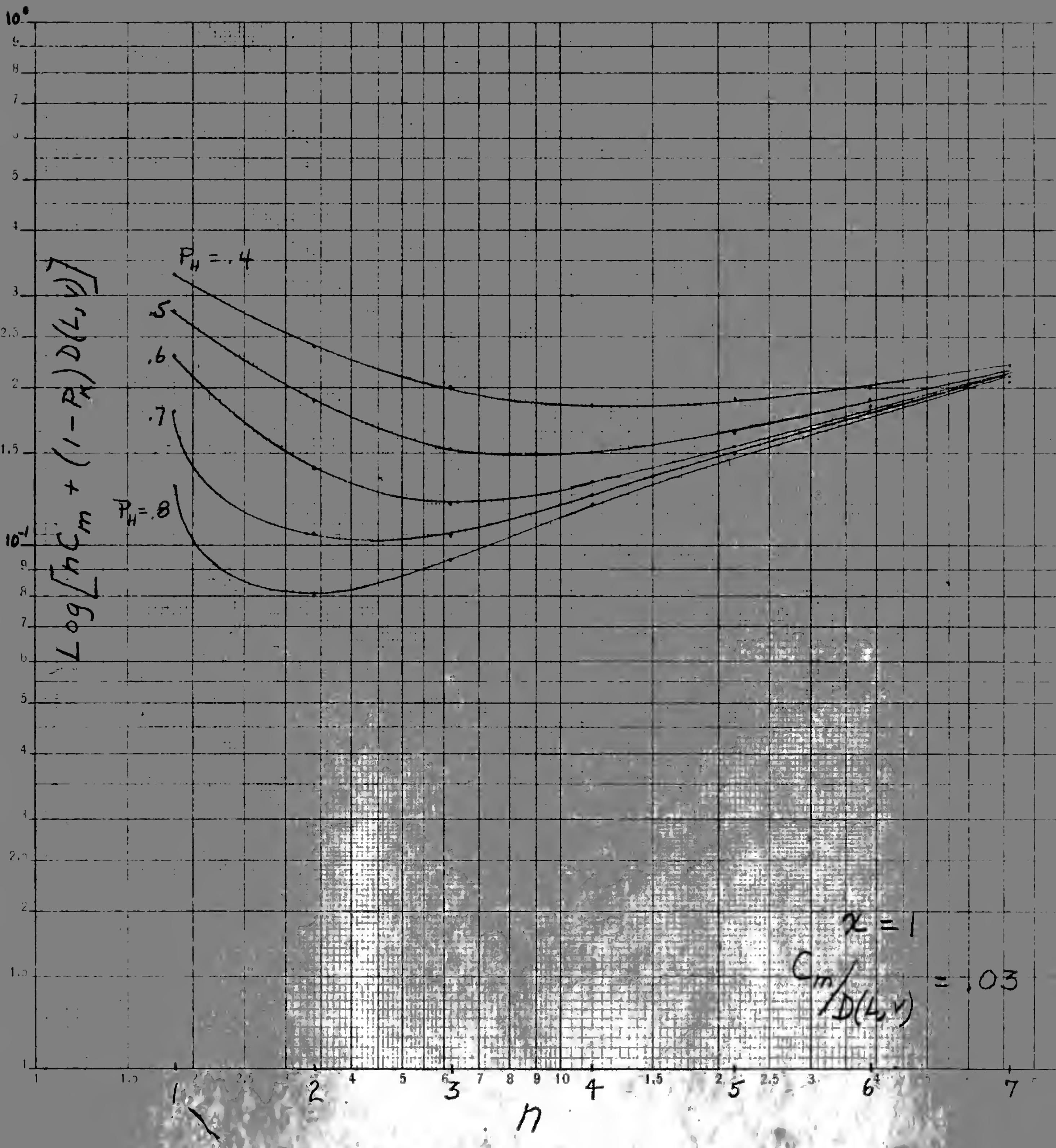

Figure 5 (c)